Dust-acoustic wave instabilities in collisional plasmas

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Current-driven dust-acoustic wave instabilities in a collisional plasma with variable-charge dusts are studied. The effects of electron and ion capture by the dust grains, the ion drag force, as well as dissipative mechanisms leading to changes in the particle numbers and momenta, are taken into account. Conditions for the instability are obtained and discussed for both weak and strong ion drag. It is shown that the threshold external electric field driving the current is relatively large in dusty plasmas because of the large dissipation rates induced by the dusts. The current-driven instability may be associated with dust cloud filamentation at the initial stages of void formation in dusty RF discharge experiments.

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I. INTRODUCTION

Collective phenomena in dusty plasmas are the subject of growing interest because dust particulates or impurities often appear [1,2] in the plasma processing chambers for manufacturing semiconductor parts. Such particles either grow naturally in or are externally introduced into the discharge. Dust grains are also observed in many space environments, such as interstellar clouds, planetary rings, cometary tails, etc. [3].

Dusty plasmas are affected by external dc electric fields, such as that in the sheath/presheath regions, or that additionally introduced for electrode biasing, ion extraction, etc. In such cases the occurrence of directed plasma flows is unavoidable. The latter can give rise to current-driven instabilities that develop at the dust-acoustic [4–6] or dust-ion-acoustic [6,7] time scales.

Electron and ion capture/release by the dust grains, usually occurring at the same time scale as that of dust charge variation, can strongly affect collective processes in dusty plasmas. A realistic description of plasma particle balance thus requires a self-consistent accounting of the particle creation and loss mechanisms such as direct and step-wise ionization, ambipolar diffusion, volume recombination, etc., which can also occur at the same time scale in many plasmas. These processes are especially important for maintaining the stationary state of the system [8,9]. It has recently been shown [10] that particle creation and loss, as well as dust-induced momentum loss of the electrons and ions, can affect the dust-ion-acoustic instability.

On the dust-acoustic time scale, the effects mentioned can affect the dispersion properties of the dust-acoustic waves [11–16]. The low-frequency dust acoustic waves were found to be unstable [17,18]. It has also been demonstrated that ion drag on the dust grains is important for the instability and can lead to the formation of regions void of dusts in the plasma [18,22–24]. In this paper, we consider the effects of plasma particle capture/release by the dusts as well as ion drag on the development of the current-driven instability at the dust-acoustic time scale. Relevance of our results to void formation, dust cloud filamentation, and instabilities in dusty RF discharges is discussed.

The paper is organized as follows: In Sec. II the problem is formulated and the basic set of equations is given. In Sec. III, the dispersion relation describing the dust-acoustic instability in an external dc electric field is derived. In Sec. IV, the real and imaginary parts of the frequency are obtained. In Secs. V and VI we analyze the instability conditions in the limit of weak and strong ion drag. Our results and their relevance to the dust void experiments are discussed in Secs. VII and VIII.

II. FORMULATION

Consider a three-component plasma in an external dc electric field $E_0\hat{x}$. The massive dust grains, containing a significant proportion of the plasma's negative charge, are treated as a charged fluid with varying average charge. The external dc electric field causes the electrons and dusts to drift with different velocities in a direction opposite to the ions. The dynamics of the particles is described by the fluid continuity and momentum equations

$$\frac{\partial n_e}{\partial t}+\frac{\partial (n_e v_e)}{\partial x}=-\nu_{ed}n_e+\mathcal{S}, \tag{1}$$

$$\nu_e^{\rm eff}v_e+\frac{T_e}{n_e m_e}\frac{\partial n_e}{\partial x}=-\frac{e}{m_e}E, \tag{2}$$

$$\frac{\partial n_i}{\partial t}+\frac{\partial (n_i v_i)}{\partial x}=-\nu_{id}n_i+\mathcal{S}, \tag{3}$$

$$v_i\frac{\partial v_i}{\partial x}+\nu_i^{\rm eff}v_i+\frac{T_i}{n_i m_i}\frac{\partial n_i}{\partial x}=\frac{e}{m_i}E, \tag{4}$$

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$$\frac{\partial v_d}{\partial t}+v_d\frac{\partial v_d}{\partial x}+\nu_{dn}v_d+\frac{T_d}{n_d m_d}\frac{\partial n_d}{\partial x}+\mu^i_{\rm drag}(v_d-v_i)$$
$$=-\frac{Z_d e}{m_d}E, \tag{5}$$

$$\frac{\partial n_d}{\partial t}+\frac{\partial(n_d v_d)}{\partial x}=0, \tag{6}$$

where E is the total electric field in the plasma including the steady-state (zero-order) field E_0, and m_e, m_i, m_d, n_i, n_e, n_d, v_i, v_e, v_d are the masses, densities, fluid velocities of the plasma electrons, ions, and dusts, respectively, T_e, T_i, and T_d are the electron, ion, and dust temperatures, and ν_{ed} and ν_{id} are the rates of capture of electrons and ions by the dust grains. We assume the dust to be cold, valid for $T_d \ll T_e, T_i$. In Eq. (5) the term $\mu^i_{\rm drag}(v_d-v_i)$ corresponds to the ion drag force acting on dust particles. We have defined $\mathcal{S}=\nu_{\rm ion}n_e-\rho n_e^2$, where $\nu_{\rm ion}$ is the ionization rate, and ρ is the coefficient of volume recombination.

For the effective electron collision frequency, we have $\nu_e^{\rm eff}=\nu_{en}+\nu_{ei}+\nu_e^{\rm el}+\nu_e^{\rm ch}$, where ν_{en} is the electron-neutral collision frequency, ν_{ei} is the frequency of electron-ion collisions, $\nu_e^{\rm el}$ is the frequency of the elastic electron-dust Coulomb collisions, and $\nu_e^{\rm ch}$ is the effective frequency of charging collisions (due to collection of plasma particles by the dusts). For the ion component we have the analogous relation $\nu_i^{\rm eff}=\nu_{in}+\nu_{ie}+\nu_i^{\rm el}+\nu_i^{\rm ch}$, where ν_{in} is the frequency of ion-neutral collisions, ν_{ie} is the rate of ion-electron collisions, respectively, $\nu_i^{\rm el}$ is the frequency of the elastic ion-dust Coulomb collisions, and $\nu_i^{\rm ch}$ is from the collection of ions by the dust grains. We note that both the elastic and charging collisions affect the momentum of the light particles. Additional terms affecting the plasma particle densities, such as step-wise ionization, ambipolar, or anomalous diffusion, etc., can be added to the right-hand sides of Eqs. (1) and (3). The charge neutrality condition $n_i=n_e+Z_d n_d$, where Z_d is the magnitude of the negative dust charge, completes the basic set of equations.

For the dust charging, we adopt the results from the well-known electrostatic probe theory [19–21]. The fluctuation of the average dust charge is then described by the charge balance equation

$$\frac{\partial \tilde{q}_d}{\partial t}+v_d\frac{\partial \tilde{q}_d}{\partial x}+\nu_d^{\rm ch}\tilde{q}_d=-|I_{e0}|\frac{\tilde{n}_e}{n_{e0}}+|I_{i0}|\frac{\tilde{n}_i}{n_{i0}}, \tag{7}$$

where

$$I_{e0}=-\pi a^2 e(8T_e/\pi m_e)^{1/2}n_{e0}\exp(e\Delta\varphi_g/T_e), \tag{8}$$

and

$$I_{i0}=\pi a^2 e(8T_i/\pi m_i)^{1/2}n_{i0}(1-e\Delta\varphi_g/T_i) \tag{9}$$

are the steady-state electron and ion currents at the dust surface, $q_{d0}=C\Delta\varphi_g$ is the stationary dust charge, $C=a(1+a/r_D)$ is the effective grain capacitance, $\Delta\varphi_g=\varphi_g-\varphi_0$ is the steady-state potential difference between the grain and the adjacent plasma, φ_0 is the floating potential, $\tilde{q}_d$ is the perturbation of the average charge, $\tilde{n}_e$ and $\tilde{n}_i$ are the variations of the electron and ion densities, respectively, and $\nu_d^{\rm ch}=\omega_{pi}^2 a\mathcal{T}/\sqrt{2\pi}V_{Ti}$ is the dust charging rate, where $\mathcal{T}=(T_e+T_i)/T_e+\varphi_d^{\rm el}/\varphi_e^{\rm th}$, $\varphi_d^{\rm el}=Z_{d0}e/a$, $\varphi_e^{\rm th}=T_e/e$, $V_{Ti}=(T_i/m_i)^{1/2}$ is the ion thermal velocity, a is the average radius of the dust particle, r_D is the plasma Debye radius, ω_{pi} is the ion plasma frequency, and the subscript 0 denotes steady-state, or zeroth-order, quantities. Since the external electric field leads to a steady ion flow $v_{i0}=eE_0/m_i\nu_i^{\rm eff}$, the temperature T_i in Eq. (9) involves both random and directed motion, and is approximately given by [20,21] $T_i\approx T_{i0}+m_i v_{i0}^2/2$, where T_{i0} is the temperature representing thermal motion. Equation (9) then covers both the thermal- and monoenergetic-ion limits often invoked in the literature on probe theories [19,20]. For the electrons, the thermal energy is much larger than the directed energy associated with the steady electron flow $v_{e0}=-eE_0/m_e\nu_e^{\rm eff}$ due to the external field, so that the effect of the latter in the electron charging equation can be ignored.

Furthermore, for the charging collision frequency we have (see, e.g., Ref. [25])

$$\nu_e^{\rm ch}=\frac{3}{2}\nu_i^{\rm ch}\frac{n_{i0}}{n_{e0}}\frac{\alpha\sigma}{T_i/T_e+\alpha}=\nu_d^{\rm ch}\frac{\alpha\gamma\sigma}{\mathcal{T}}, \tag{10}$$

where $\alpha=\mathcal{T}-1$, $\gamma=(Z_{d0}n_{d0}/n_{e0})\varphi_e^{\rm th}/\varphi_d^{\rm el}$, and $\sigma=4+\varphi_d^{\rm el}/\varphi_e^{\rm th}$. The rate of electron and ion capture by the grain is given by

$$\nu_{ed}=\frac{n_{i0}}{n_{e0}}\nu_{id}=\nu_d^{\rm ch}\frac{\alpha\gamma}{\mathcal{T}}, \tag{11}$$

and the rates of elastic electron- and ion-dust Coulomb collisions are

$$\nu_e^{\rm el}=\nu_i^{\rm el}\alpha\frac{T_i}{T_e}\frac{n_{i0}}{n_{e0}}\exp\left(\frac{\varphi_d^{\rm el}}{\varphi_e^{\rm th}}\right)=\frac{2}{3}\nu_d^{\rm ch}\frac{\alpha\gamma\Lambda}{\mathcal{T}}\exp\left(\frac{\varphi_d^{\rm el}}{\varphi_e^{\rm th}}\right), \tag{12}$$

where $\Lambda=\ln(r_D/a)$ is the Coulomb logarithm and $a\ll r_D$. The electron- and ion-neutral collision frequencies are $\nu_{en}=N_n\sigma_{en}V_{Te}$, $\nu_{in}=N_n\sigma_{in}V_{Ti}$, where N_n is the neutral gas density, σ_{en} and σ_{in} are the electron (ion)-neutral collision cross sections, V_{Te} is electron thermal speed [26]. The expressions for the electron-ion and ion-electron collision frequencies are given in Ref. [19].

In Eq. (5) there are two dissipative terms which affect the dust dynamics, namely $\nu_{dn}v_d$, where

$$\nu_{dn}\sim 4m_n N_n a^2 V_{Tn}/m_d$$

is the frequency of dust-neutral collisions where m_n, N_n, and V_{Tn} are mass, density, and thermal velocity of the neutrals, respectively. The other dissipative term is $\mu^i_{\rm drag}(v_d-v_i)$, where

$$\mu^i_{\rm drag}\sim 4m_i n_{0i}b^2 V_S/m_d,$$

is the ion drag coefficient, $b\sim a\sqrt{\pi}(1-\Delta\varphi_g/T_i)$ is the ion-collection impact parameter [21], and V_S is the ion-acoustic velocity. Equations (1)–(7) together with the charge neutrality condition describe the dusty plasma system.

III. DISPERSION RELATION

In the steady state, we have $n_{e0}=(\nu_{\rm ion}-\nu_{ed})/\rho$ and $n_{i0}=(\nu_{ed}/\nu_{id})n_{e0}$ for the equilibrium electron and ion densities, and

$$v_{d0}=-\frac{Z_{d0}eE_0(1-1/\mathcal{A})}{m_d(\nu_{dn}+\mu^i_{\rm drag})} \quad (13)$$

for the stationary dust drift velocity. We have defined $\mathcal{A}=Z_{d0}m_i\nu_i^{\rm eff}/m_d\mu^i_{\rm drag}$. Note that since $\nu_{(i,e)}^{\rm eff}>(\nu_{en},\nu_{in})$ the magnitudes of the electron and ion drift velocities are lower than that under the constant charge approximation [4]. The second term in Eq. (13) is responsible for the pushing of the dust particles in a direction opposite to the electric force acting on the negatively charged dusts. Thus, if the ion drag is weak, the dusts drift in the negative *x* direction, and if the ion drag is sufficiently strong, they drift in the opposite direction. In particular, if $\mathcal{A}>1$, the negatively charged dust particles drift in the same direction as the plasma electrons. Otherwise the dusts and electrons drift in the opposite directions. A combination of strong ion drag and the ionization instability has been invoked as a possible cause of voids in dusty RF discharges [18,23,24].

We linearize Eqs. (1)–(7) and assume that the perturbed quantities depend on *x* and *t* like $\exp[i(kx-\omega t)]$. We then obtain for the electron density perturbation

$$\tilde{n}_e=-kn_{e0}e\tilde{E}/\eta_e m_e\nu_e^{\rm eff}, \quad (14)$$

where $\eta_e=\Omega_{e0}+i(\nu_{\rm ion}-\nu_{ed}+k^2V_{Te}^2/\nu_e^{\rm eff})$, and $\Omega_{e0}=\omega-kv_{e0}$. For the ion density perturbation, we have

$$\tilde{n}_i=kn_{i0}e\tilde{E}/\eta_i m_i\nu_i^{\rm eff}, \quad (15)$$

where $\eta_i=\Omega_{i0}+i(\nu_{id}+k^2V_{Ti}^2/\nu_i^{\rm eff})$, $\Omega_{i0}=\omega-kv_{i0}$. For the perturbed dust density, we have

$$\tilde{n}_d=-i\frac{kn_{d0}eZ_{d0}}{m_d\Omega_{d0}[\Omega_{d0}+i(\nu_{dn}+\mu^i_{\rm drag})]}\times\left[\tilde{E}\left(1-\frac{\Omega_{i0}+i\nu_{id}}{\eta_i\mathcal{A}}\right)+E_0\frac{\tilde{Z}_d}{Z_{d0}}\right], \quad (16)$$

where $\Omega_{d0}=\omega-kv_{d0}$.

From Eqs. (7), (14), and (15) we obtain

$$\tilde{Z}_d=-i\frac{k|I_{e0}|\tilde{E}}{\eta_e m_e\nu_e^{\rm eff}(\Omega_{d0}+i\nu_d^{\rm ch})}\left(1+\frac{\eta_e}{\eta_i}\frac{m_e}{m_i}\frac{\nu_e^{\rm eff}}{\nu_i^{\rm eff}}\right) \quad (17)$$

for the variation of dust charge. The expressions for the fluid velocities of the plasma particles can be derived from Eqs. (2), (4), and (5).

Together with the quasineutrality condition $\tilde{n}_i=\tilde{n}_e+n_{d0}\tilde{Z}_d+Z_{d0}\tilde{n}_d$, the expressions (14)–(17) lead to the dispersion relation

$$\frac{\omega_{pe}^2}{\eta_e\nu_e^{\rm eff}}\left(1+\frac{\nu_{ed}}{\nu_d^{\rm ch}}\right)+\frac{\omega_{pi}^2}{\eta_i\nu_i^{\rm eff}}\left(1+\frac{n_{e0}\nu_{ed}}{n_{i0}\nu_d^{\rm ch}}\right)+\frac{i\omega_{pd}^2}{\Omega_{d0}[\Omega_{d0}+i(\nu_{dn}+\mu^i_{\rm drag})]}\left(1-\frac{\Omega_{i0}+i\nu_{id}}{\eta_i\mathcal{A}}\right)=0, \quad (18)$$

where we have defined $\Omega_{e0}=\Omega_{d0}-k\vartheta_e$, $\Omega_{i0}=\Omega_{d0}-k\vartheta_i$, $\vartheta_e=v_{e0}-v_{d0}$, and $\vartheta_i=v_{i0}-v_{d0}$.

In the limiting case of negligible dust-specific electron and ion dissipation (by formally setting $\nu_{(i,e)}^{\rm eff}\to\nu_{(i,e)n}$ and $\nu_{(i,e)d}\to 0$), dust-charge variations ($\nu_d^{\rm ch}\to 0$), as well as ion drag force ($\mu^i_{\rm drag}\to 0$), the dispersion relation (18) is reduced to (7) of Ref. [4].

IV. FREQUENCY OF THE UNSTABLE MODE

The dispersion relation (18) can be analyzed numerically for any set of physical parameters. However, analytical solutions can be obtained in the case when the equilibrium density of the ions much exceeds that of the electrons ($n_{i0}\gg n_{e0}$). This may happen at the initial stage of the filamentary mode, when the charge density $Z_{d0}n_{d0}$ of the dusts constitutes a significant proportion of total plasma negative charge density.

Letting $\Omega_d=\Omega_d'+i\Omega_d''$ and separating the real and imaginary parts of Eq. (18), we obtain

$$\Omega_d''=-\frac{\nu_{dn}}{2}\left[1+\mathcal{B}\left(1-\frac{k\mu_i}{\Omega_d'}\right)+\frac{\mu^i_{\rm drag}}{\nu_{dn}}\frac{k\vartheta_i}{\Omega_d'}\right] \quad (19)$$

for the imaginary part of the frequency in a dust frame. We have defined $\mathcal{B}=Z_{d0}m_i\nu_i^{\rm eff}/m_d\nu_{dn}$. One can see from Eq. (19) that there is no instability if $k\vartheta_i<\Omega_d'$. If, however, the relative ion-to-dust drift in the external electric field is such that $\vartheta_i>\Omega_d'/k$, instability of the dust-acoustic waves becomes possible. The conditions for the instability will be discussed in detail below. The corresponding real part Ω_d' of the frequency is given by

$$\Omega_d'^2=\Omega_d''(\Omega_d''+\nu_{dn})-\mu^i_{\rm drag}\nu_{id}+\frac{m_iZ_{d0}}{m_d}(\nu_i^{\rm eff}\Omega_d''+\nu_{id}\nu_i^{\rm eff}+k^2V_{Ti}^2), \quad (20)$$

and near marginal stability ($\Omega_d''\approx 0$) we can write

$$\Omega_d'=\Omega_{d0}'\left[1+\frac{\nu_{id}\mu^i_{\rm drag}}{(\Omega_{d0}')^2}(\mathcal{A}-1)\right]^{1/2}, \quad (21)$$

where $\Omega_{d0}'=(Z_{d0}m_i/m_d)^{1/2}kV_{Ti}$ is the corresponding frequency of the dust-acoustic waves in low-collisional plasmas [11]. If the effects of ion capture/release by the dusts can be neglected (which is formally achieved by equating ν_{id} to zero), Ω_d' coincides with Ω_{d0}'.

The condition for the instability strongly depends on the direction of the stationary dust drift in the external electric field (13). As already mentioned, we see that the dust grains drift in the direction of the electric force (opposite to the ion drift) when the inequality $\mathcal{A}>1$ is satisfied. This means that the effect of the steady electric force on the dusts overcomes that of the ion drag force. If the opposite inequality holds, the dust grains are pushed by the ion drag force and move in the same direction with the ions. Note that the inequality varies continuously with growing dust grains [18]. It is possible that $\mathcal{A}>1$ is satisfied for initially small grains, and when the grains becoming sufficiently large the condition becomes eventually violated. We shall therefore consider these cases separately.

V. WEAK ION DRAG

First, we consider the situation when the electric force acting on the dusts is dominant, that is, when $\mathcal{A}>1$. It can be shown that ϑ_i must exceed the threshold value

$$\vartheta_i^{\text{thres}}=\frac{\Omega_d'}{k}\left(1+\frac{1}{\mathcal{B}}\right)\left(1-\frac{1}{\mathcal{A}}\right)^{-1}$$

for the realization of the instability.

From Eq. (21) it follows that the real part of the frequency in the frame moving with the drifting dusts is larger than the frequency Ω_{d0}' of dust-acoustic waves in ideal plasmas. Note that in this case $v_{d0}<0$, and $\Omega_d'>0$, and the relative ion-to-dust drift is always positive ($\vartheta_i>0$) since the ions and dusts drift in the opposite directions.

When $\mathcal{A}\gg 1$, the expression for the threshold ion-to dust drift can be approximated as

$$\mu_i^{\text{thresh}}=\frac{\nu_{dn}}{k}\left(\frac{m_d}{m_iZ_{d0}}\right)^{1/2}(1+\mathcal{B})\left[\frac{\nu_{id}}{\nu_i^{\text{eff}}}+\left(\frac{kV_{Ti}}{\nu_i^{\text{eff}}}\right)^2\right]^{1/2}$$

and further simplifications can be obtained for large or small (compared with unity) $(\nu_i^{\text{eff}}/\nu_{dn})(m_iZ_{d0}/m_d)$. In the same limiting case, from Eq. (19) we obtain that the external electric field must exceed the threshold

$$E_0^{\text{thresh}}\approx\frac{m_i}{e}\nu_i^{\text{eff}}\frac{\Omega_d'}{k}(1+\mathcal{B})\left(\mathcal{B}-\frac{\mu_{\text{drag}}^i}{\nu_{dn}}\right)^{-1} \tag{22}$$

to achieve the dust-acoustic instability, where Ω_d' can be approximated by Eq. (21).

We can compare the threshold values of the external electric field in the present case (E_0^{thresh}) and in the case ($\mathcal{E}_0^{\text{thresh}}$) when the effects of ion collection by the dusts as well as ion-dust Coulomb and charging collisions are neglected. Namely,

$$\frac{E_0^{\text{thresh}}}{\mathcal{E}_0^{\text{thresh}}}\sim\frac{1+\nu_i^{\text{eff}}Z_{d0}m_i/\nu_{dn}m_d}{1+\nu_{in}Z_{d0}m_i/\nu_{dn}m_d}\sqrt{1+\frac{\nu_{id}\nu_i^{\text{eff}}}{k^2V_{Ti}^2}}, \tag{23}$$

which shows that the instability threshold is higher here. The expression (23) can be simplified for small and large $\mathcal{B}$.

VI. STRONG ION DRAG

We now consider the situation when the ion drag force acting on the dust particles is stronger than the external electric force, or $\mathcal{A}<1$. In this case the ion drag force overcomes the electric force and pushes dust particles in the direction of the ion drift ($v_{d0}>0$ and $v_{i0}>0$). Here, the frequency of the unstable modes in a dust frame $\Omega_d'=\omega-kv_{d0}$ can be either positive or negative. Since the ions and dusts drift in the same direction, the relative ion-to-dust drift is smaller than in the weak ion drag case. For the magnitude of the relative ion-to-dust drift we can write

$$\vartheta_i=\frac{eE_0}{m_i\nu_i^{\text{eff}}}\left(1-\frac{\mu_{\text{drag}}^i}{\nu_{dn}+\mu_{\text{drag}}^i}|\mathcal{A}-1|\right)$$

so that it remains positive, since the absolute value of the dust drift velocity is $\mu_{\text{drag}}^i/(\nu_{dn}+\mu_{\text{drag}}^i)$ times smaller than that of the ion drift.

Unstable solutions can be obtained only if the dust drift velocity exceeds the phase velocity of the dust acoustic waves ($v_{d0}>\omega/k$), and $\vartheta_i>\vartheta_i^{\text{thresh}}$, where

$$\vartheta_i^{\text{thresh}}=\frac{|\Omega_d'|}{k}\left(1+\frac{1}{\mathcal{B}}\right)\left|1-\frac{1}{\mathcal{A}}\right|^{-1} \tag{24}$$

and we note that in this case $\Omega_d'<0$.

An interesting feature in this case is that the absolute value of the real part of the frequency (21) is less than Ω_{d0}' and it decreases with increasing ion drag force. This means that the real part of the frequency can vanish, providing the existence of non-oscillating solutions. From Eq. (21) one sees that in the case of strong ion drag oscillating solutions can exist only if the inequality

$$\left|1-\frac{1}{\mathcal{A}}\right|\frac{\nu_{id}\nu_i^{\text{eff}}}{k^2V_{Ti}^2}<1 \tag{25}$$

is satisfied. This inequality can be violated for long-wavelength perturbations and sufficiently strong ion collisions. In this case realization of aperiodic instability (with no real part of the frequency) is possible. A similar situation has been reported for the ionization instability of long-wavelength dust acoustic waves affected by strong ion drag [22].

VII. APPLICATION

Examining the conditions for the current-driven dust-acoustic instability, one can see that the terms in Eq. (19) leading to the instability are proportional to the effective frequency of *ion* collisions. This means that ion collisions have a destabilizing effect on the dust acoustic waves. Electron-neutral collisions in a dust-free plasma lead to resistive ion-acoustic instability [27] in a similar manner. It is important to note that competition between the electrostatic and ion drag forces on dust particles strongly affects the conditions for the instability. The ratio of these forces is $\mathcal{A}$, which can be greater or less than unity. In fact, as shown below, this ratio has a rather broad range.

The conditions for the instability strongly depend on the value of $\mathcal{B}$. If it is small compared with unity but still large compared with $\mu_{\text{drag}}^i/\nu_{dn}$, then the relative ion-to-dust drift should be large such that $\vartheta_i^{\text{thresh}}\sim\Omega_d'/k\mathcal{B}\gg\Omega_d'/k$. In this case we face the situation that the destabilizing effect of ion collisions is small, but the effect of the ion drag is even smaller. If the opposite inequality, $\mathcal{B}\gg 1$, holds, ϑ_i should simply exceed Ω_d'/k, as is the case for the constant dust charge case [4]. This conclusion is expected since ion drag then becomes unimportant.

Assuming the typical values $Z_{d0}\sim 10^3$–10^4, $m_i/m_d \sim 10^{-13}$–10^{-11}, $\nu_i^{\rm eff}\sim 10^6\,{\rm s}^{-1}$, $n_i\sim 10^8$–$10^9\,{\rm cm}^{-3}$, $N_n \sim 10^{13}$–$10^{15}\,{\rm cm}^{-3}$, $a\sim 1$–$5\,\mu$m, and $T_e/T_i\sim T_i/T_n\sim 10$, we found that the parameter $\mathcal{A}$ can be in the range 10^{-4}–2.5×10^4. For plasmas with an ionization degree of the order 10^{-4}, and similar values of the other parameters the ratio $\mathcal{B}$ in the expressions for the thresholds of the ion-to-dust drift and the electric field is approximately $10-10^3$ times less than the one above. One expects that situations with low values of $\mathcal{B}$ are more typical in experiments with micron size particles, and the value ϑ_i should be much larger than Ω_d'/k.

Using the parameters of existing experiments (with different sizes and charges of the dust particles), we shall show that both cases considered in Secs. V and VI are possible. The strong ion drag case will be discussed in more detail as it is related to the dust-void experiments [18].

For conditions representative of the experiments in Refs. [4], [14], and [15], namely $m_i/m_e\sim 51\,600$, $m_i/m_d\sim 4.7\times 10^{-14}$, $n_{i0}\sim 10^9\,{\rm cm}^{-3}$, $\nu_{in}\sim 8.8\times 10^4\,{\rm s}^{-1}$, $N_n\sim 3\times 10^{15}\,{\rm cm}^{-3}$, $T_e\sim 3$ eV, $T_i\sim 0.1$ eV, $T_n\sim 0.0125$ eV, and $a \sim 5\,\mu$m, we obtain $\mathcal{A}\sim 10^2$. Therefore, the effect of ion drag can be neglected and the instability seems to follow the scenario given in Sec. V. Similar conclusion on the unimportance of ion drag in the above case has been made by D'Angelo and Merlino [4]. On the other hand, one also obtains $\mathcal{B}\sim 10^{-1}$. Thus, ϑ_i must exceed $10\Omega_d'/k$ for the dust-acoustic wave instability to be realized.

Now we turn our attention to the dust-void experiments [18]. We have assumed that the electric field is externally applied and homogeneous. Clearly, from the Poisson equation it follows that electric fields sufficiently strong for the development of the instability can be generated because of charge fluctuations in the pristine dusty plasmas. Namely, for $n_{e0}\sim 10^{10}$–$10^{12}\,{\rm cm}^{-3}$, $\tilde{n}_e/n_{e0}\sim 10^{-3}$–$10^{-4}$, and characteristic filament size $L\sim 1$ cm, an electric field $E_0 \sim 1.5$–150 V/cm can be generated. Measurements near the instability threshold showed $E_0\sim 20$ V/cm [18].

An important feature of the experiment of Ref. [18] is the relatively high operating gas pressure (400 mTorr), providing a large neutral gas density $N_n\sim 1.4\times 10^{16}\,{\rm cm}^{-3}$. For such an operating regime the assumption of the volume recombination controlled regime would seem to be valid [28]. For lower operating gas pressures the diffusion terms in particle balance Eqs. (1) and (3) should be taken into account. The high density of neutrals yields a relatively large rate $\nu_{in} \sim 10^7\,{\rm s}^{-1}$ of ion-neutral collisions in argon at $p_0 = 400$ mTorr. This rate appears larger than the frequencies of ion-dust Coulomb ($\nu_i^{\rm el}$) and charging ($\nu_i^{\rm ch}$) collisions. For $E_0\sim 20$ V/cm, $T_e\sim 3$ eV, and $T_i\sim 0.05\,T_e$, the characteristic ion drift velocity is $v_{i0}\sim 4.4\times 10^4$ cm/s. For this value of v_{i0} and the spatial scale ~ 1 cm of the filamentation, the nonlinear term $v_i\partial v_i/\partial x$ in Eq. (4) is ignorable. However, this is valid only at the initial stage of the instability. At later stages the nonlinear term can be crucial in determining self-organized nonlinear dissipative structures of the dust void [24]. Furthermore, near the filamentation threshold, assuming $a\sim 0.13\,\mu$m we obtain for the dust mass $m_d\sim 9.2\times 10^{-15}$ g. From the condition of the equality of the equilibrium electron and ion grain currents we obtain $Z_{d0}\sim 250$. For $n_d\sim 10^8\,{\rm cm}^{-3}$ and $T_n\sim 0.1\,T_i$, we obtain $\nu_{dn}\sim 1.31\times 10^3\,{\rm s}^{-1}$ for the dust-neutral collision frequency. The latter exceeds that of the characteristic oscillations associated with plasma striation at the initial stage of the instability [18].

There is a fairly large uncertainty in determining the ion drag force, which depends strongly on the impact parameter b for ion collection in the ion drag coefficient. Assuming a local potential difference $\Delta\varphi_g\sim 3$ eV, and $T_i\sim 0.15$ eV, for the ion drag coefficient we have $\mu_{\rm drag}^i\sim 1.65\times 10^3\,{\rm s}^{-1}$ so that $\mu_{\rm drag}^i/\nu_{dn}\sim 1.26$. For argon gas and 130 nm dusts the key parameters are: $Z_{d0}m_i/m_d = 1.95\times 10^{-6}$, $\mathcal{B} = 0.015$, and $\mathcal{A} = 0.01$, respectively. This means that the effect of the ion drag on the current-driven dust-acoustic instability can really be important for the development of the filamentary mode in dust void experiments [18].

Furthermore, the inequality (25) can be violated and aperiodic instabilities can arise. Similar results have been reported earlier for the ionization instability affected by ion drag [22]. This is consistent with the observed sudden onset of the instability [18].

In our investigation the ionization terms in the particle balance equations are electron density dependent. We have also accounted for the variation of the ionization rate with $\tilde{n}_e$, and fluctuations in the temperature have been neglected. This is consistent with the statement that the enhanced ionization rate can be attributed to a higher electron density rather than T_e in the initial perturbation [18]. Accounting for the variations of the ionization cross-section with $\tilde{E}$ would have lead us to a similar ionization instability considered by D'Angelo [22].

The sudden onset of the instability can be understood if we note that the threshold value of the electric field, which can easily be derived from Eq. (24), appears to scale like $E_0^{\rm thresh}\sim 1/a$. This means that for small dust grains the instability threshold cannot be reached. When the grains grow in size, the threshold is greatly decreased and instability onset becomes possible [29].

VIII. DISCUSSION

In the above we have assumed $n_{i0}\gg n_{e0}$ for estimating the instability growth rate (19). It should be noted however that dust void formation is a dynamical process in which the dust particles are pushed out of regions where the initial fluctuations of the electric field appear. This can lead to an increase of the electron density and the above inequality may eventually be violated.

We have also assumed that the neutral background is stationary in the steady state and is not perturbed in the initial stage of the instability. Perturbation of the neutral density occurs because of ionization and recombination. However, these perturbations are of the order of $\tilde{n}_e$, and their relative effect, characterized by the ratio $\tilde{N}_n/N_{n0}\sim \tilde{n}_e/N_{n0}$, is small since $n_{e0}\ll N_{n0}$. The neutrals affect the motion of the dusts because of neutral drag. The latter can easily be incorporated into the basic equations [25].

In our study, a simple expression $\beta_i \approx e/m_i \nu_i^{\rm eff}$ for the ion mobility was used. The latter relates the ion drift velocity v_{i0} to the external electric field E_0. If the electric field is strong, the ion mobility can become a nonlinear function of E_0, tending to $\beta_i \sim \sqrt{E_0}$ for $E_0/p_0 \gg \kappa_0$, where p_0 is the operating gas pressure, and κ_0 depends on the type of the operating gas [30]. In this case, our results will be somewhat modified.

It is worth emphasizing that for the cases considered in Secs. V and VI the relative ion-to-dust drift is always positive ($\vartheta_i > 0$). It can be much larger in the small drag case when the ions and dusts drift in opposite directions. Thus, the current-driven instability is generally stronger in the weak ion drag case for the same external electric field.

In our model, which accounts for the ionization and recombination processes, it is possible to self-consistently determine the equilibrium electron and ion densities in dusty gas discharge plasmas. For self-consistency we consider the plasma as a thermodynamically open system [31] and included the particle capturing processes into the electron and ion conservation equations. We have also shown that in the intermediate pressure regime a stationary state of the plasma cannot be achieved without accounting for the ionization and volume recombination processes. Further improvements of the model can be made by including other transport mechanisms [28] typical for RF discharge plasmas and may take place on the same time scale as that of ionization and dust charge relaxation. Since ionization, diffusion, and recombination are all density dependent, electrostatic phenomena in the system can be strongly affected. Thus, the presence of dusts can affect the entire discharge system through a modification of the ionization-recombination-diffusion balance.

It should also be mentioned that care should be taken in using Eq. (6) describing conservation of the dust particles. In cases when $n_d m_d \sim n_i m_i$, a corresponding source responsible for the conservation of total dust and ion mass density should be added [32]. Such a situation can occur, e.g., in dusty interstellar clouds [33].

IX. CONCLUSION

A theory of the current-driven dust-acoustic instability in an external dc electric field is presented. Conditions for the instability are discussed. Our self-consistent model accounts for electron and ion capture/release by the dusts, dust-enhanced Coulomb elastic and charging inelastic collisions, as well as ionization-recombination balance of electrons and ions. It is found that the conditions for the instability are very different for weak and strong ion drag. In general, the relative ion-to-dust drift should exceed a threshold value for the unstable modes to be excited. For strong ion drag and long-wavelength perturbations, it appears possible to realize a nonpropagating unstable mode. We have also shown that the threshold of the external electric field is larger for variable-charge dusts compared to that for constant dust charge because of the large dissipation rate induced by the dusts. The results presented may be useful in explaining the nature of the filamentation instability, which takes place at the initial stage of the formation of stable dust voids.

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